Kinematics—Projectiles

1. A volleyball hit into the air has an initial speed of 10 meters per second. Which vector best represents the angle above the horizontal that the ball should be hit to remain in the air for the greatest amount of time?

Base your answers to questions 2 through 4 on the information below.

A projectile is fired from the ground with an initial velocity of 250 meters per second at an angle of 60° above the horizontal.

\[ 250 \text{ m/s} = 5 \text{ cm} \]

2. On the diagram at right, use a protractor and ruler to draw a vector to represent the initial velocity of the projectile. Begin the vector at P, and use a scale of 1.0 centimeter = 50 meters per second.

3. Determine the horizontal component of the initial velocity.

\[ V_{ix} = V_i \cos \Theta = \frac{250}{5} \cos 60° \approx 125 \text{ m/s} \]

4. Explain why the projectile has no acceleration in the horizontal direction. [Neglect friction.]

\[ a_x = 0 \text{ m/s}^2, \text{ because gravity does not work sideways.} \]

5. Two stones, A and B, are thrown horizontally from the top of a cliff. Stone A has an initial speed of 15 meters per second and stone B has an initial speed of 30 meters per second. Compared to the time it takes stone A to reach the ground, the time it takes stone B to reach the ground is

1. the same
2. twice as great
3. half as great
4. four times as great

6. The diagram below represents the path of a stunt car that is driven off a cliff, neglecting friction.

\[ V_{ix} = V_{Aix} = V_{Bix} \]

(does not change)

Compared to the horizontal component of the car's velocity at point A, the horizontal component of the car's velocity at point B is

1. smaller
2. greater
3. the same
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Base your answers to questions 7 through 9 on the information and diagram below.

An object was projected horizontally from a tall cliff. The diagram below represents the path of the object, neglecting friction.

7. How does the magnitude of the horizontal component of the object's velocity at A compare with the magnitude of the horizontal component of the object's velocity at point B?
   
   Same

8. How does the magnitude of the vertical component of the object's velocity at point A compare with the magnitude of the vertical component of the object's velocity at point B?
   
   Less

9. On the diagram above, sketch a likely path of the horizontally projected object, assuming that it was subject to air resistance. (Diagram above)

10. A golf ball is propelled with an initial velocity of 60 meters per second at 37° above the horizontal. The horizontal component of the golf ball's initial velocity is
    1. 30 m/s
    2. 36 m/s
    3. 40 m/s
    4. 48 m/s

   \[ V_{ix} = V_i \cos \theta \]
   
11. A golf ball is hit with an initial velocity of 15 meters per second at an angle of 35° above the horizontal. What is the vertical component of the golf ball's initial velocity?
    1. 8.6 m/s
    2. 9.8 m/s
    3. 12 m/s
    4. 15 m/s

   \[ V_{iy} = V_i \sin \theta \]

12. As shown in the diagram below, a student standing on the roof of a 50-meter-high building kicks a stone at a horizontal speed of 4 meters per second.

   \[
   d_y = \frac{1}{2} a t^2 \quad \text{\(V_{iy} = 0\)}
   
   -50 = \frac{1}{2} \left( -9.81 \right) t^2
   
   t = \sqrt{10.25}
   
   t \approx 3.19 s

13. A machine launches a tennis ball at an angle of 25° above the horizontal at a speed of 14 m/s. The ball returns to level ground. Which combination of changes must produce an increase in time of flight of a second launch?
    1. decrease the launch angle and decrease the ball's initial speed
    2. decrease the launch angle and increase the ball's initial speed
    3. increase the launch angle and decrease the ball's initial speed
    4. increase the launch angle and increase the ball's initial speed

   \[ V_i \sin \theta \]
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Base your answers to questions 14 through 16 on the information below.

A kicked soccer ball has an initial velocity of 25 meters per second at an angle of 40° above the horizontal, level ground. [Neglect friction.]

14. Calculate the magnitude of the vertical component of the ball’s initial velocity. [Show all work, including the equation and substitution with units.]

\[ V_{iy} = V_i \sin \theta = \frac{25 \text{ m/s}}{5} \sin 40^\circ = 16 \text{ m/s} \]

15. Calculate the maximum height the ball reaches above its initial position. [Show all work, including the equation and substitution with units.]

\[ A \rightarrow B \quad V_{fy} = V_{iy} + 2a_y D_y \\
(0 \text{ m/s})^2 = (16 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)D_y \]

\[ D_y = 13 \text{ m} \]

16. On the diagram, sketch the path of the ball’s flight from its initial position at point P until it returns to level ground.

17. Two spheres, A and B, are simultaneously projected horizontally from the top of a tower. Sphere A has a horizontal speed of 40 m/s and sphere B has a horizontal speed of 20 m/s. Which statement best describes the time required for the spheres to reach the ground and the horizontal distance they travel?

1. Both spheres hit the ground at the same time and at the same distance from the base of the tower.
2. Both spheres hit the ground at the same time, but sphere A lands twice as far as sphere B from the base of the tower.
3. Both spheres hit the ground at the same time, but sphere B lands twice as far as sphere A from the base of the tower.
4. Sphere A hits the ground before sphere B, and sphere A lands twice as far as sphere B from the base of the tower.

Base your answers to questions 18 and 19 on the information below.

An outfielder throws a baseball to second base at a speed of 19.6 m/s and an angle of 30° above the horizontal.

18. Which pair represents the initial horizontal velocity \((v_x)\) and initial vertical velocity \((v_y)\) of the baseball?

1. \(v_x = 17.0 \text{ m/s} \) and \(v_y = 9.80 \text{ m/s}\)
2. \(v_x = 9.80 \text{ m/s} \) and \(v_y = 17.0 \text{ m/s}\)
3. \(v_x = 19.4 \text{ m/s} \) and \(v_y = 5.90 \text{ m/s}\)
4. \(v_x = 19.6 \text{ m/s} \) and \(v_y = 19.6 \text{ m/s}\)

19. If the ball was caught at the same height from which it was thrown, calculate the amount of time the ball was in the air, showing all work including the equation and substitution with units.

\[ A \rightarrow B \quad V_{fy} = V_{iy} + a_y t_{up} \]

\[ 0 \text{ m/s} = 9.80 \text{ m/s}^2 + (-9.81 \text{ m/s}^2)t_{up} \]

\[ t_{up} = 2.00 \text{ s} \]
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20. Four projectiles, A, B, C, and D, were launched from, and returned to, level ground. The data table below shows the initial horizontal speed, initial vertical speed, and time of flight for each projectile.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Initial Horizontal Speed (m/s)</th>
<th>Initial Vertical Speed (m/s)</th>
<th>Time of Flight (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40.0</td>
<td>29.4</td>
<td>6.00</td>
</tr>
<tr>
<td>B</td>
<td>60.0</td>
<td>19.6</td>
<td>4.00</td>
</tr>
<tr>
<td>C</td>
<td>50.0</td>
<td>24.5</td>
<td>5.00</td>
</tr>
<tr>
<td>D</td>
<td>80.0</td>
<td>19.6</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Which projectile traveled the greatest horizontal distance? [Neglect friction.]

1. A
2. B
3. C
4. D

21. Four identical projectiles are launched with the same initial speed, \( v \), but at various angles above the level ground. Which diagram represents the initial velocity of the projectile that will have the largest total horizontal displacement? [Neglect air resistance.]

Base your answers to questions 22 and 23 on the information and diagram below.

A child kicks a ball with an initial velocity of 8.5 meters per second at an angle of 35° with the horizontal, as shown. The ball has an initial vertical velocity of 4.9 meters per second and a total time of flight of 1.0 second. [Neglect air resistance.]

22. What is the horizontal component of the ball’s initial velocity?

\[ V_{ix} = V_i \cos \theta = (8.5 \text{ m/s}) \cos 35° \]

\[ = 7.0 \text{ m/s} \]

23. Find the maximum height reached by the ball.

\[ A \rightarrow B \]

\[ V_{fy}^2 = V_{iy}^2 + 2a_y D_y \]

\[ (0 \text{ m/s})^2 = (4.9 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2) D_y \]

\[ D_y = \frac{-24.01 \text{ m}^2}{-19.62 \text{ m/s}^2} \]

\[ D_y = 1.2 \text{ m} \]
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Base your answers to questions 24 through 26 on the information below.

The path of a stunt car driven horizontally off a cliff is represented in the diagram at right. After leaving the cliff, the car falls freely to point A in 0.50 second and to point B in 1.00 second.

24. Determine the magnitude of the horizontal component of the velocity of the car at point B. [Neglect friction.]

\[ V_x = \frac{D_x}{t} = \frac{16 - 8.0 \text{ m}}{1.00 - 0.50 \text{ s}} \]

\[ v_x = 16 \text{ m/s} \]

25. Determine the magnitude of the vertical velocity of the car at point A.

\[ V_{fy} = V_{iy} + a_{y} t \]

\[ V_{fy} = 0 \text{ m/s} + (-9.81 \text{ m/s}^2)(0.50 \text{ s}) = -4.91 \text{ m/s} \]

26. Calculate the magnitude of the vertical displacement, \( d_y \), of the car from point A to point B. [Neglect friction. Show all work, including the equation and substitution with units.]

\[ d_y = \overline{V}_y \cdot t = (\frac{-7.37 \text{ m/s}}{2})(0.50 \text{ s}) \]

\[ d_y = -1.84 \text{ m} \]

A projectile is launched into the air with an initial speed of \( v_i \) at a launch angle of 30° above the horizontal. The projectile lands on the ground 2.0 seconds later.

27. On the diagram, sketch the ideal path of the projectile.

28. How does the maximum altitude of the projectile change as the launch angle is increased from 30° to 45° above the horizontal? [Assume the same initial speed, \( v_i \).]

\[ D_y = \frac{V_i^2 \sin^2 \theta}{19.62 \text{ m/s}^2} \]

\[ (0 \text{ m})^2 = (V_i \sin 30^\circ)^2 + 2(-9.81 \text{ m/s}) D_y \]

\( D_y \) increases \( (\sin \theta \) increases \)

29. How does the total horizontal distance traveled by the projectile change as the launch angle is increased from 30° to 45° above the horizontal? [Assume the same initial speed, \( v_i \).]

\[ D_x = V_i x \cdot t \] (it increases, because 45° gives you the maximum horizontal distance)
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Base your answers to questions 30 through 32 on the information and graph below.

A machine fired several projectiles at the same angle, $\theta$, above the horizontal. Each projectile was fired with a different initial velocity, $v_i$. The graph below represents the relationship between the magnitude of the initial vertical velocity, $v_{iy}$, and the magnitude of the corresponding initial velocity, $v_i$, of these projectiles.

30. Determine the magnitude of the initial vertical velocity of the projectile, $v_{iy}$, when the magnitude of its initial velocity, $v_i$, was 40 meters per second.

From the graph at $v_i = 40 \text{ m/s}$

we get $v_{iy} = 25 \text{ m/s}$.

31. Determine the angle, $\theta$, above the horizontal at which the projectiles were fired.

$\sin \theta = \frac{v_{iy}}{v_i}$

$\theta = \sin^{-1}\left(\frac{25}{40}\right) = 39^\circ$ above the horizontal.

32. Calculate the magnitude of the initial horizontal velocity of the projectile, $v_{ix}$, when the magnitude of its initial velocity, $v_i$, was 40 meters per second. [Show all work, including the equation and substitution with units.]

$v_i^2 = v_{ix}^2 + v_{iy}^2$

$(40 \text{ m/s})^2 = v_{ix}^2 + (25 \text{ m/s})^2$

$v_{ix} = 31 \text{ m/s}$

33. A plane flying horizontally above Earth's surface at 100 meters per second drops a crate. The crate strikes the ground 30 seconds later. What is the magnitude of the horizontal component of the crate's velocity just before it strikes the ground? [Neglect friction.]

1. 0 m/s
2. 100 m/s
3. 294 m/s (does not change)
4. 394 m/s

34. The diagram below represents the path of an object after it was thrown.

What happens to the object's acceleration as it travels from A to B?

1. It decreases.
2. It increases.
3. $g$ does not change.

35. A 0.2-kilogram red ball is thrown horizontally at a speed of 4 meters per second from a height of 3 meters. A 0.4-kilogram green ball is thrown horizontally from the same height at a speed of 8 meters per second. Compared to the time it takes the red ball to reach the ground, the time it takes the green ball to reach the ground is

1. one-half as great
2. twice as great
3. the same
4. four times as great

are irrelevant for time to

36. A ball is thrown at an angle of 38° to the horizontal. What happens to the magnitude of the ball's vertical acceleration during the total time interval that the ball is in the air?

1. It decreases, then increases.
2. It decreases, then remains the same.
3. It increases, then decreases.
4. It remains the same.
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Base your answers to questions 37 through 39 on the information and diagram below.

A projectile is launched horizontally at a speed of 30 meters per second from a platform located a vertical distance \( h \) above the ground. The projectile strikes the ground after time \( t \) at horizontal distance \( d \) from the base of the platform. [Neglect friction.]

37. On the diagram above, sketch the theoretical path of the projectile. (solid line above)

38. Calculate the horizontal distance, \( d \), if the projectile’s time of flight is 2.5 seconds. [Show all work, including the equation and substitution with units.]

\[
\begin{align*}
D_x &= V \cdot t \\
&= (30, \text{ m/s}) (2.5 \text{ s}) \\
&= 75 \text{ m}
\end{align*}
\]

39. Express the projectile’s total time of flight, \( t \), in terms of the vertical distance, \( h \), and the acceleration due to gravity, \( g \). [Write an appropriate equation and solve it for \( t \).]

\[
h = \frac{1}{2} gt^2
\]

\[
\frac{2h}{g} = t^2 \\
t = \sqrt{\frac{2h}{g}}
\]

40. A ball is thrown horizontally at a speed of 24 meters per second from the top of a cliff. If the ball hits the ground 4.0 seconds later, approximately how high is the cliff?

1. 6.0 m
2. 39 m
3. 78 m
4. 96 m

\[
h = \frac{1}{2} (9.81 \text{ m/s}^2) (4.0 \text{ s})^2
\]

41. A golf ball is given an initial speed of 20 meters per second and returns to level ground. Which launch angle above level ground results in the ball traveling the greatest horizontal distance? [Neglect friction.]

1. 60°
2. 45°
3. 30°
4. 15°

42. A golf ball is hit at an angle of 45° above the horizontal. What is the acceleration of the golf ball at the highest point in its trajectory? [Neglect friction.]

1. 9.8 m/s² upward
2. 9.8 m/s² downward
3. 6.9 m/s² horizontal
4. 0.0 m/s²

43. A soccer player kicks a ball with an initial velocity of 10 meters per second at an angle of 30° above the horizontal. The magnitude of the horizontal component of the ball’s initial velocity is

1. 5.0 m/s
2. 8.7 m/s
3. 9.8 m/s
4. 10 m/s

44. A vector makes an angle, \( \theta \), with the horizontal. The horizontal and vertical components of the vector will be equal in magnitude if angle \( \theta \) is

1. 30°
2. 45°
3. 60°
4. 90°
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45. The diagram below represents a setup for demonstrating motion.

When the lever is released, the support rod withdraws from ball B, allowing it to fall. At the same instant, the rod contacts ball A, propelling it horizontally to the left. Which statement describes the motion that is observed after the lever is released and the balls fall? [Neglect friction.]
1. Ball A travels at constant velocity.
2. Ball A hits the tabletop at the same time as ball B.
3. Ball B hits the tabletop before ball A.
4. Ball B travels with an increasing acceleration.

46. A projectile launched at an angle of 45° above the horizontal travels through the air. Compared to the projectile's theoretical path with no air friction, the actual trajectory of the projectile with air friction is
1. lower and shorter
2. lower and longer
3. higher and shorter
4. higher and longer

47. A projectile is fired with an initial velocity of 120 meters per second at an angle, \( \theta \), above the horizontal. If the projectile's initial horizontal speed is 55 meters per second, then angle \( \theta \) measures approximately
1. 13°
2. 27°
3. 63°
4. 75°

\( \cos \theta = \frac{55}{120} \), \( \theta = \cos^{-1} \left( \frac{55}{120} \right) \)

48. A baseball is thrown at an angle of 40° above the horizontal. The horizontal component of the baseball's initial velocity is 12 meters per second. What is the magnitude of the ball's initial velocity?
1. 7.71 m/s
2. 9.20 m/s
3. 15.7 m/s
4. 18.7 m/s

\( \sqrt{v^2} = \frac{12 \text{ m/s}}{\cos 40°} \approx 15.7 \text{ m/s} \)

49. A toy rocket is launched twice into the air from level ground and returns to level ground. The rocket is first launched with initial speed \( v \) at an angle of 45° above the horizontal. It is launched the second time with the same initial speed, but with the launch angle increased to 60° above the horizontal. Describe how both the total horizontal distance the rocket travels and the time in the air are affected by the increase in launch angle. [Neglect friction.]

\( D_x \) decreases (for 45° \( D_x \) is max)
while \( t_{flight} \) increases (for 90° \( t_{flight} \) is max)