A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in flight and air resistance is negligible. ('94)

a. Determine the time it takes for the ball to reach the plane of the fence.

\[ d_x = v_{ix} t = (v_i \cos \theta) t \]
\[ t = \frac{d_x}{v_i \cos \theta} = \frac{32 \text{ m}}{(20 \text{ m/s}) \cos 37^\circ} \approx 2.05 \text{ s} \]

b. Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

\[ d_y = v_{iy} t + \frac{1}{2} a_y t^2 = (v_i \sin \theta) (t) + \frac{1}{2} (-g) t^2 \]
\[ d_y = (20 \text{ m/s} \sin 37^\circ)(2.05) + \frac{1}{2} (-9.80 \text{ m/s}^2)(2.05 \text{ s})^2 \]
\[ d_y = 4.4 \text{ m} \text{, so the ball will clear the fence by 1.9 m (4.4 m - 2.5 m = 1.9 m)} \]
\[ V_{ix} = V_i \cos \theta = (20 \text{ m/s}) \cos 37^\circ = 16 \text{ m/s} \]
\[ V_{iy} = V_i \sin \theta = (20 \text{ m/s}) \sin 37^\circ = 12 \text{ m/s} \]
\[ V_{fx} = V_{ix} = \frac{16 \text{ m/s}}{5} \]

(c) On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.

\[ V_{fy} = V_{iy} + a_y t \]
\[ V_{fy} = 12 - 9.8 t \]

2. A particle is fired from the ground with an initial velocity \( v_0 \) at an angle \( \theta_0 \) above the horizon.

(a) How would you combine the two equations for vertical and horizontal displacements to find the equation of the path?

\[ V_{ix} = v_0 \cos \theta_0 \]
\[ V_{ix} = \frac{D_x}{t} \]
\[ t = \frac{D_x}{V_{ix}} = \frac{D_x}{v_0 \cos \theta_0} \]

\[ D_y = V_{iy} t + \frac{1}{2} a_y t^2 = \left( v_0 \sin \theta_0 \right) \left( \frac{D_x}{v_0 \cos \theta_0} \right) + \frac{-g}{2} \left( \frac{D_x}{v_0 \cos \theta_0} \right)^2 \]

\[ D_y = \left( \tan \theta \right) D_x - \frac{1}{2v_0^2 \cos^2 \theta_0} D_x^2 \]

or

\[ y = b x - a x^2 \]
b. What is the shape of the trajectory of the particle?

A downward parabola shifted toward the right.

\[ v_y = v_{yi} + at_y \]

\[ v_y(0) = 0 \]

\[ 0 = v_0 \sin \theta_0 + (-g) t_{up} \]

\[ t_{flight} = \frac{2v_0 \sin \theta_0}{g} \]

c. What is the time of flight of the particle?

\[ t_{up} = t_{down} = \frac{1}{2} t_{flight} \]

\[ t_{up} = \frac{v_0 \sin \theta_0}{g} \]

d. What is the maximum height reached by the particle?

\[ D_y = v_{yi} t + \frac{1}{2} (-g) t^2 \]

where

\[ t = t_{up} = \frac{v_0 \sin \theta_0}{g} \]

\[ D_y = \left( v_0 \sin \theta_0 \right) \left( \frac{v_0 \sin \theta_0}{g} \right) = \frac{g}{2} \left( \frac{v_0 \sin \theta_0}{g} \right)^2 \]

\[ D_y = \frac{1}{2} \frac{v_0^2 \sin^2 \theta_0}{g} \]

e. What is the range of the particle?

\[ D_x = v_{ix} \cdot t \]

where

\[ t = t_{flight} = \frac{2v_0 \sin \theta_0}{g} \]

\[ D_x = (v_0 \cos \theta_0) \cdot \frac{2v_0 \sin \theta_0}{g} \]

\[ D_x = \frac{v_0^2 \sin 2\theta_0}{g} \]

f. For what value of \( \theta_0 \) does the particle travel the furthest?

For \( \theta = 45^\circ \), \( 2\theta = 90^\circ \) and \( \sin 2\theta_0 = 1 \),

which means:

\[ D_{x \max} = \frac{v_0^2}{g} \]